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A Modern Approach to Pricing For Risk

Stephen J Mildenhall

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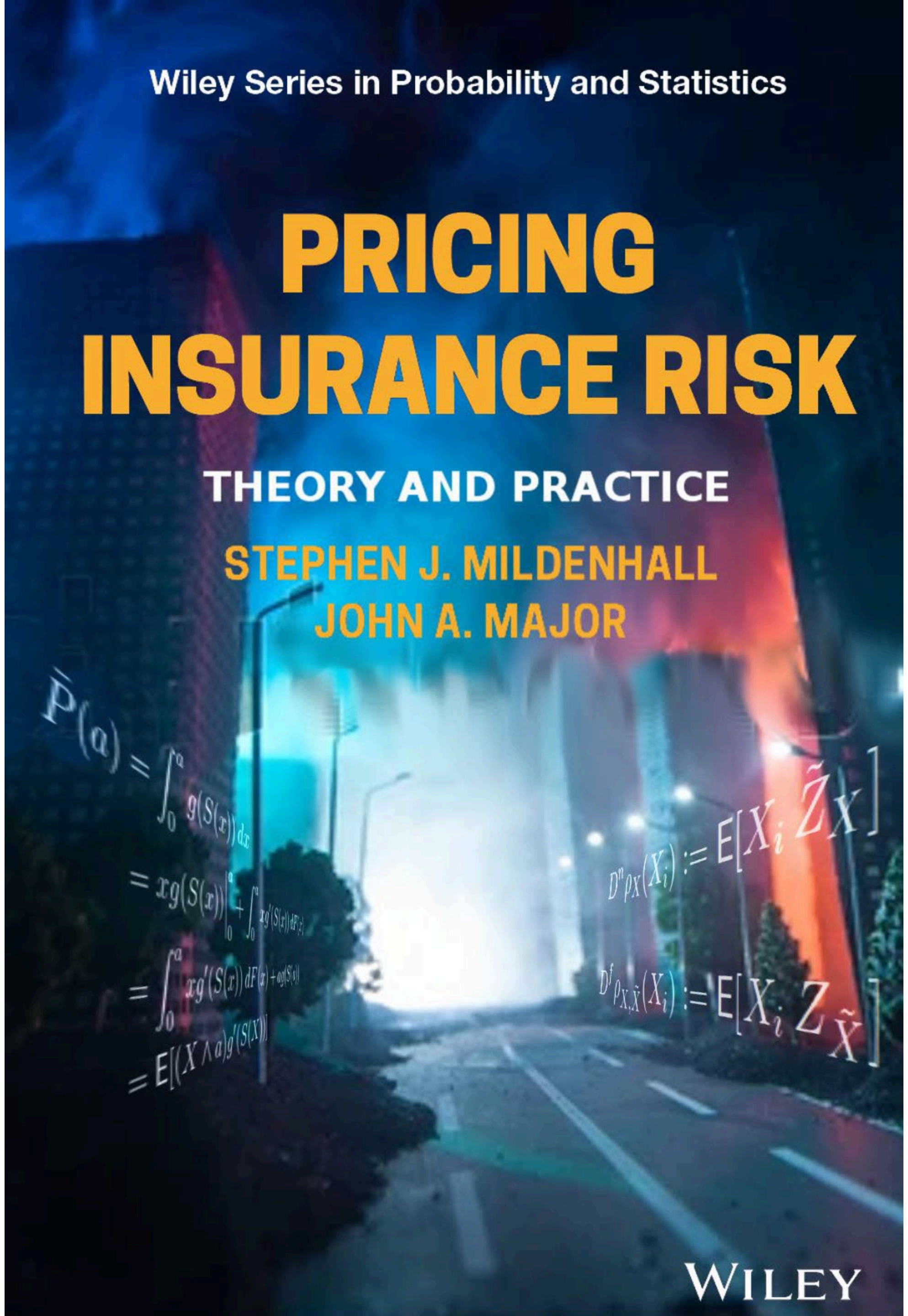
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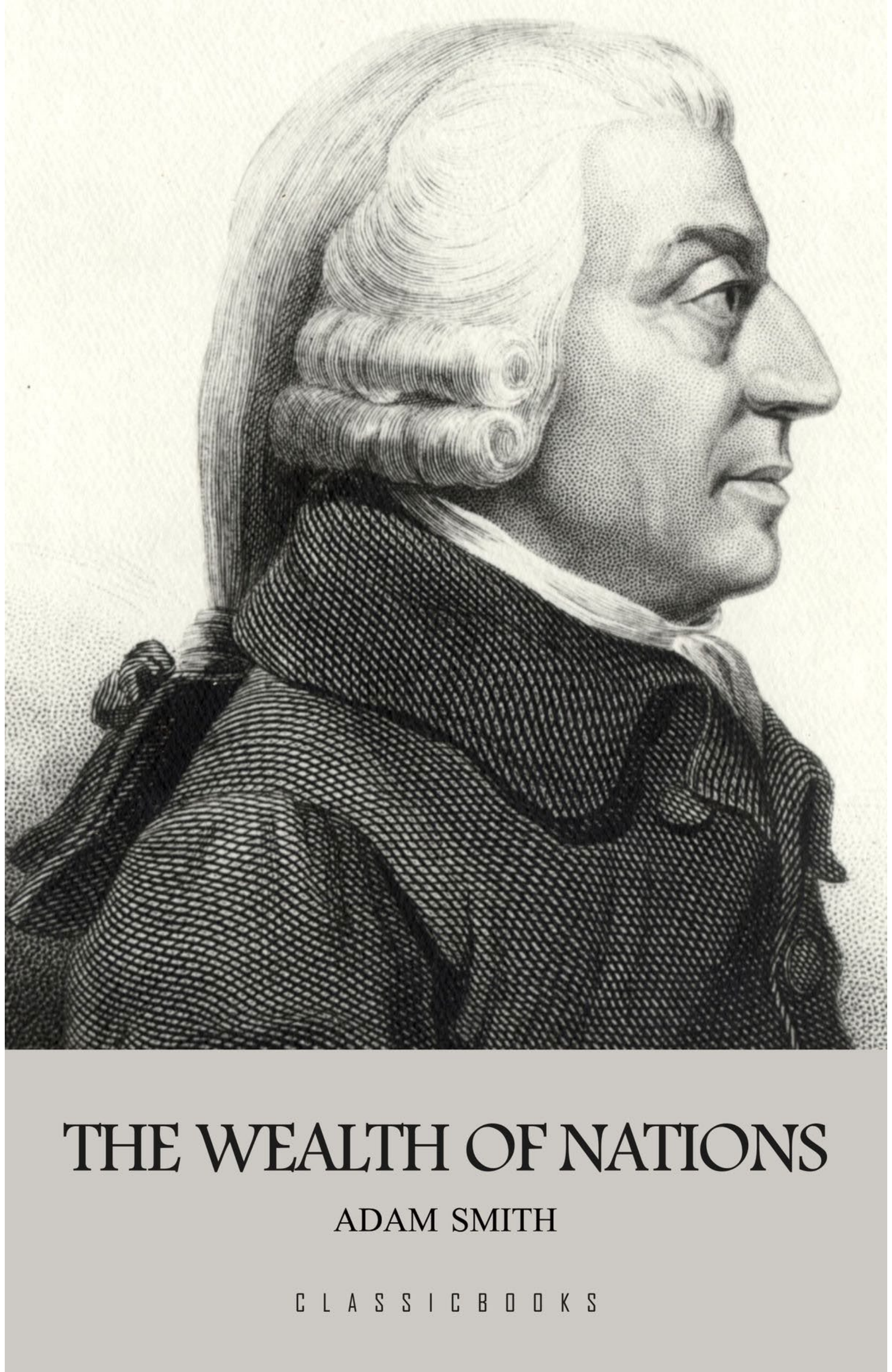
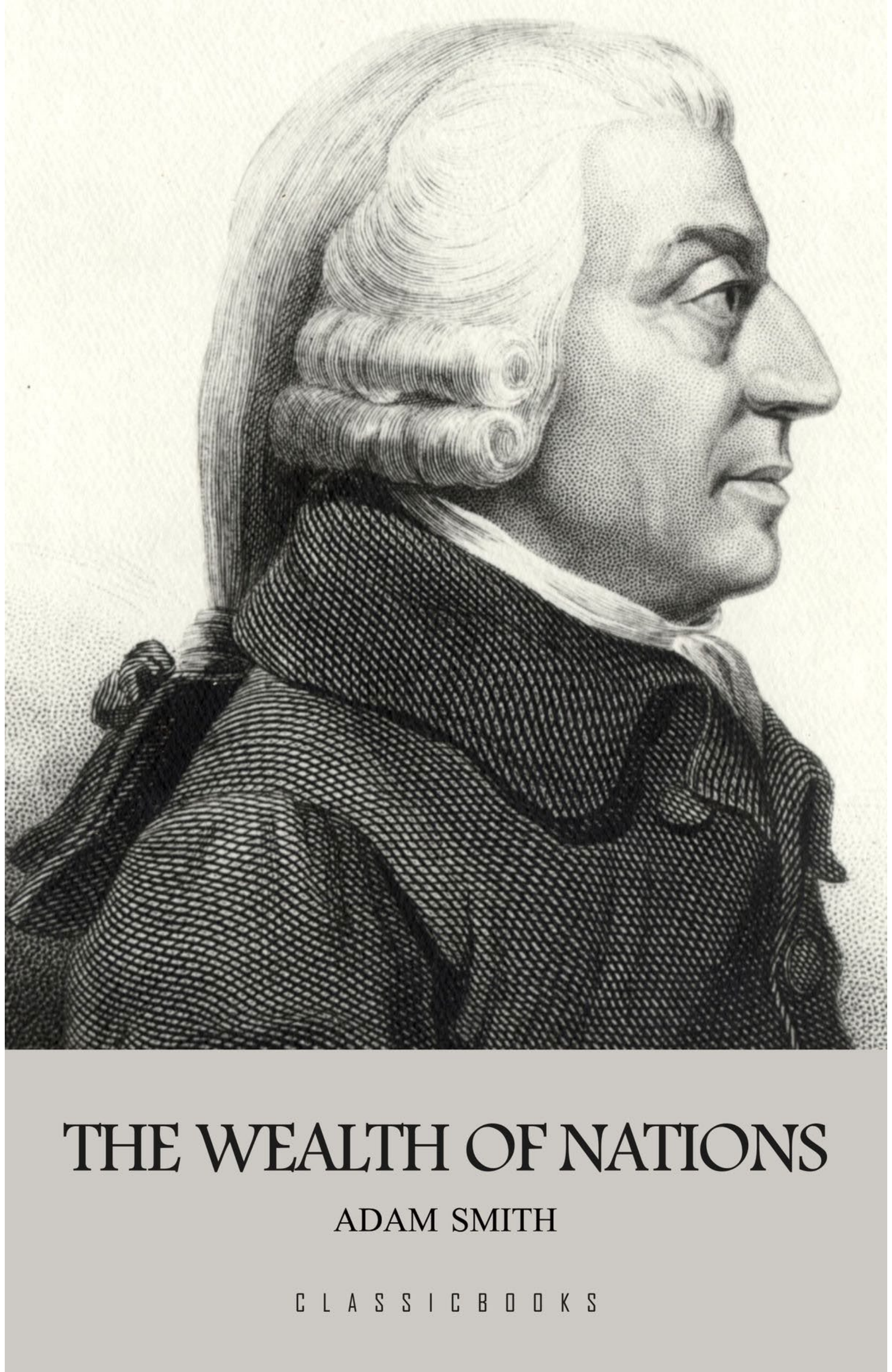
Avoid Arbitrary Assumptions



In order to make insurance a trade at all, the common premium must be sufficient to compensate the common losses, to pay the expense of management, and to afford such a profit as might have been drawn from an equal capital employed in any common trade.

Book 1, Ch X, Part I, 5th Edition, 1789



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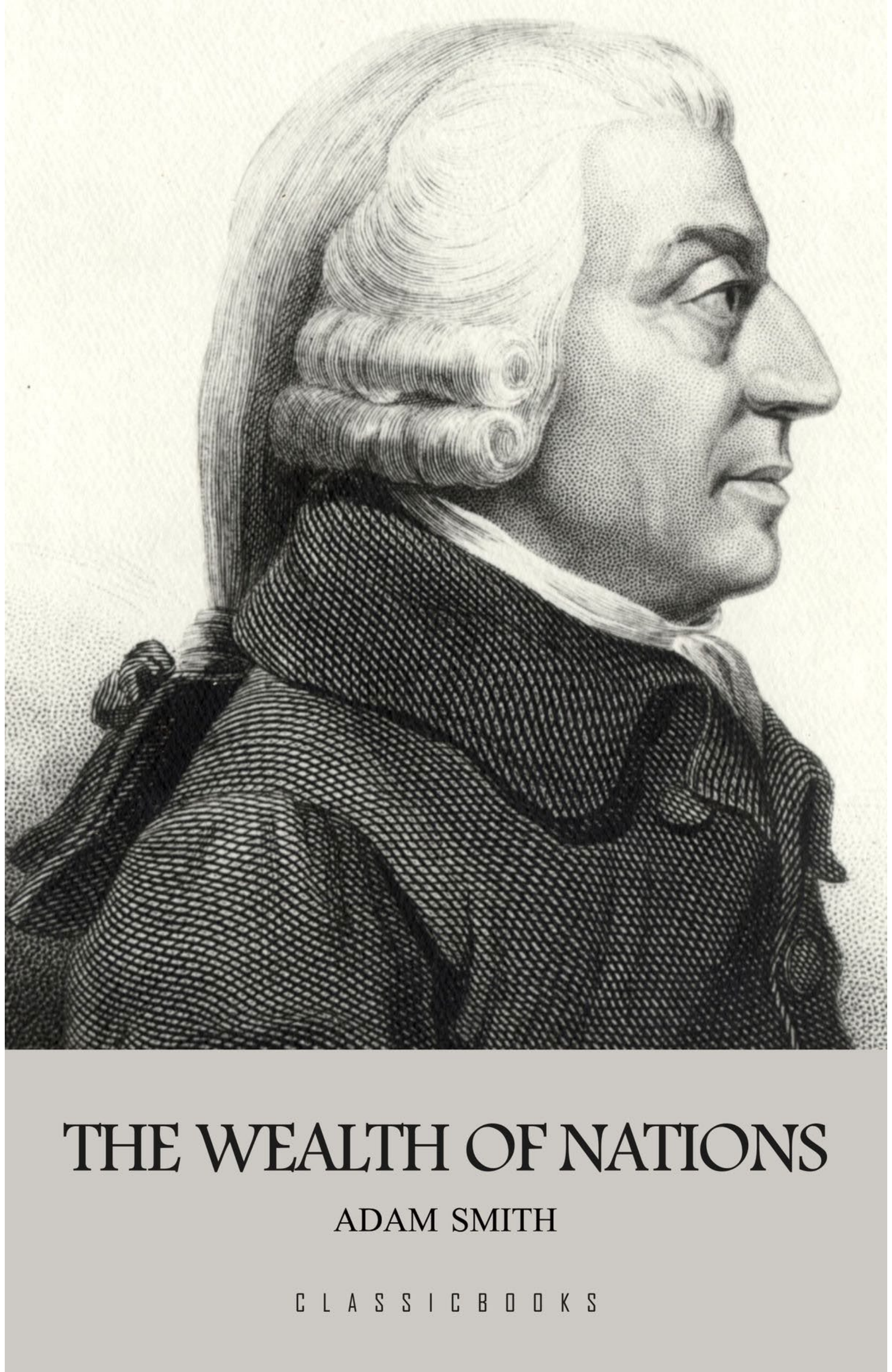

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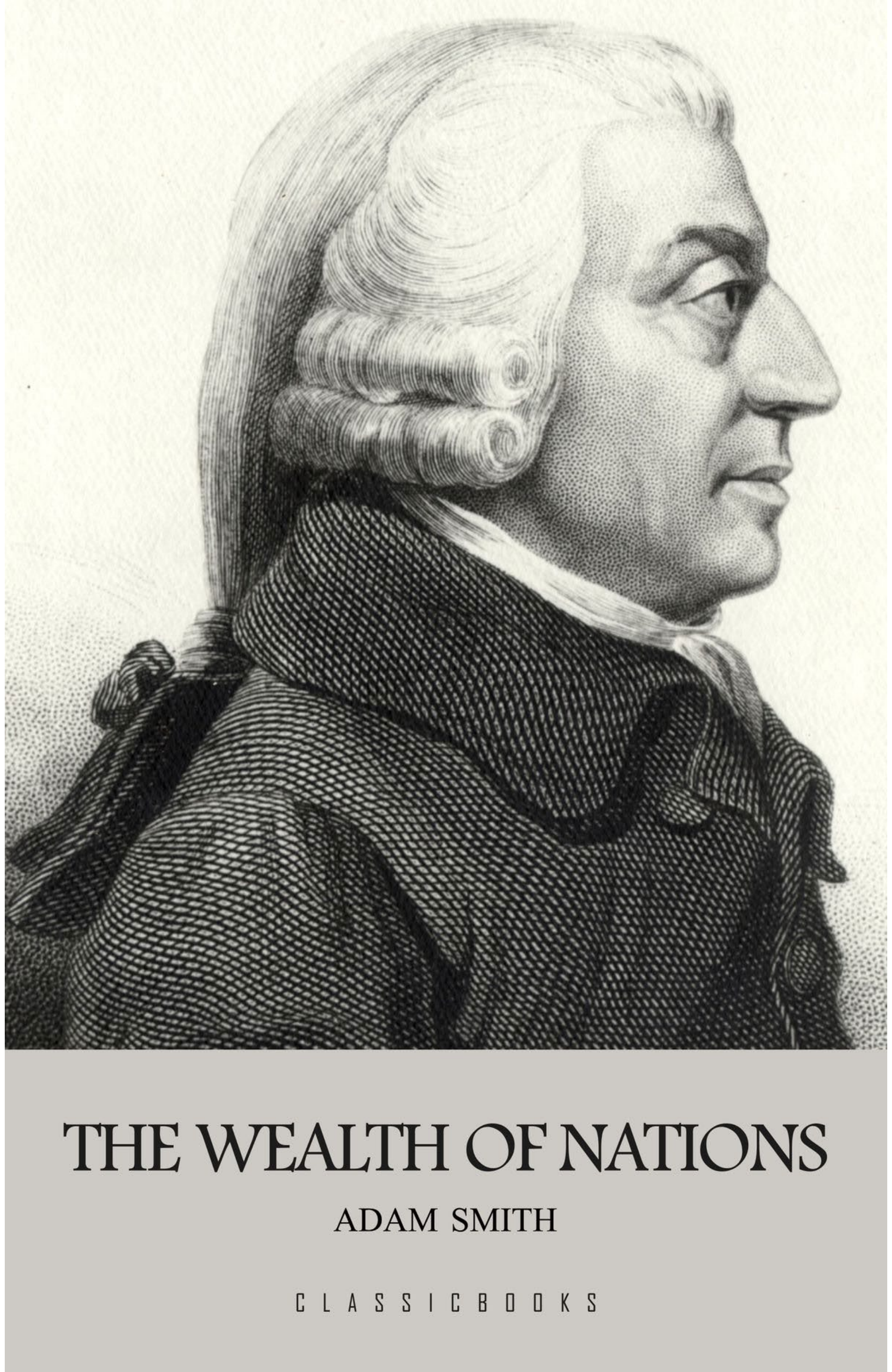

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Margin = ($ Amount of Capital) x (Cost / $)

Two perspectives on Cost / $

1. User → Cost of Capital
2. Provider → Target Return or Target **Risk** Return

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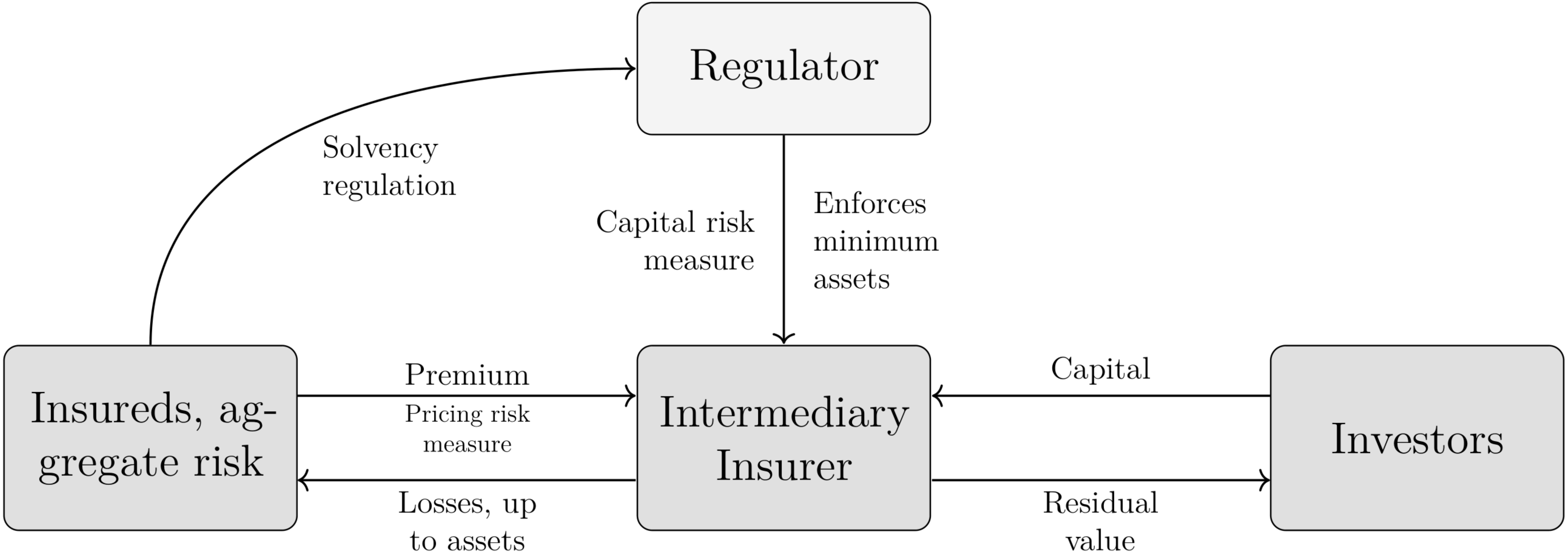
Whose opinions matter?



Avoid Arbitrary Assumptions

- **Portfolio pricing**
  - Regulator [customers, rating agency, market norms] determines total **amount** of capital
  - Investors determine **cost** of capital
  - Mutuals: policyholder funds like (lower-cost) equity
- **Price allocation to units**
  - Marginal Euler allocation
  - [Decomposition, co-TVaR, co-measure] **natural** allocation
  - Generally the same!
  - Both depend on an unspecified aggregate **pricing functional**

Portfolio Regulatory Capital Standards

- **Proxy one-year aggregate VaR**
- Agg VaR = expected + Occ VaR
- Factor based RBC + cat charge + diversification
- Challenging technical properties

- No one cares about your internal capital model
- They care a lot about your assessment of catastrophe risk
- Management should focus on **confidence in pricing**

Finance 101 fails: price and motivation

- H(ot) = pays 1 if average temperature in August is above 90^{o}F
- Objective probability of loss = *p*
- Premium "functional" P(H)
- Underwriter wants **positive** margin, P(H) > *p*

- C(old) = 1 – H, pays 1 if average temperature below 90^{o}F
- Objective probability of loss = 1 – *p*
- Since 1 = H + C, no-arbitrage & an **additive** functional implies P(1) = 1 = P(H) + P(C)
- → Cold has **negative** margin

Additive pricing and no arbitrage

- Writing H and C creates a riskless portfolio H + C = 1, value 1
- Insur**ed** initiated
  - Write H for A(H)
- Insur**er** initiated
  - Write C for B(C)
  - Creates hedge portfolio
  - Need to find buyer
  - Market for lemons
- A(H) has a positive margin
- B(C) has a negative margin because buyer skeptical
- No arbitrage: A(H) + B(C) = 1

The Bid-Ask Spread for a Security/Policy X

- Transaction: if I sell X to you and...
  - if you want to buy X, then you pay my **asking price**
  - if I want to sell X, then I receive your **bid price**

Motivation alters price

Bid ≤ Ask

Ask(X) = –Bid(–X)

- Sell X (insur**ed** initiated): A(X)
- Sell –X (insur**er** initiated): B(–X)
- Hedged portfolio X – X ≡ 0 must have value 0 by no arbitrage
- A(X) + B(–X) = 0
- → A(X) = –B(–X)

The Bid-Ask Spread for Insurers

- Insureds want to **buy** insurance
- Pay **asking** price
- Insurers want to **sell** securities
- Receive **bidding** price

Insurers pool and transform **bought** policies into **sold** securities…motivation alters price

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Bernoulli Risks: The Fundamental Atoms of Risk

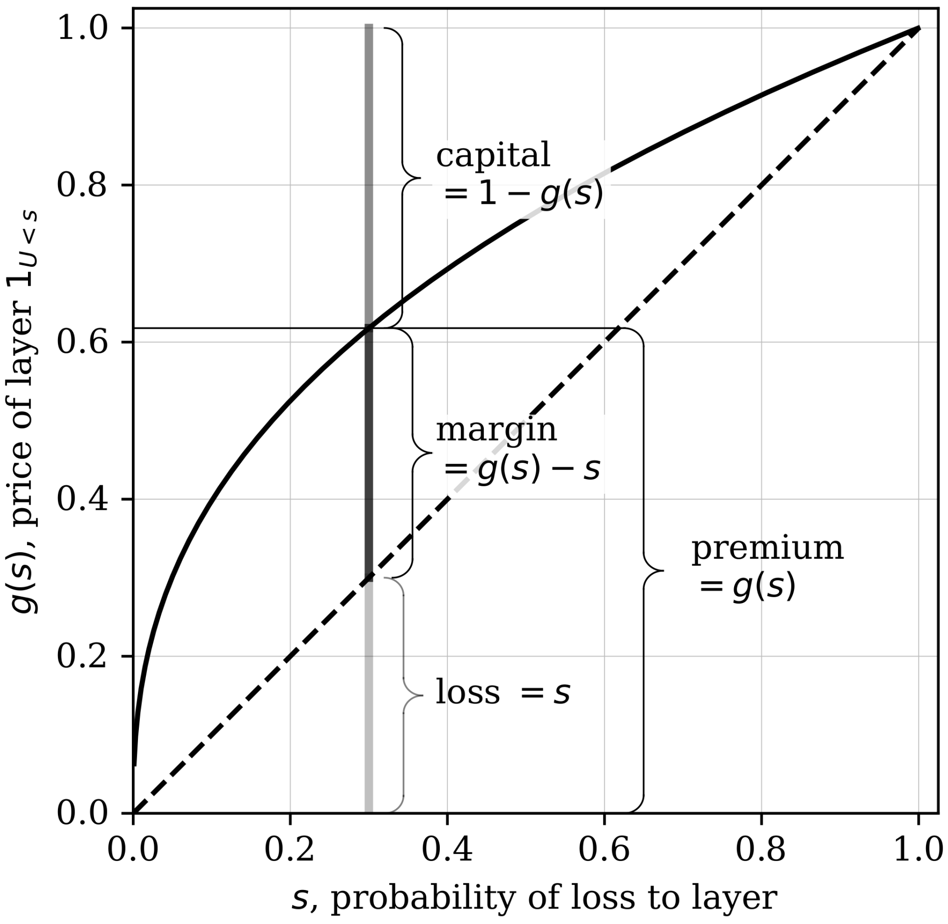
- **Bernoulli risk** (security)
  X = Ber(*p*) pays $1, probability *p*
- Premium **law invariant**
- Insurer asks premium A(X) = g(*p*) to write Ber(*p*), depends only on *p*
- Insurer supported by assets 1

- Insurer raises capital by selling residual 1 – X = Ber(1 – *p*)
- Investors bid B(1 – X) for residual
- B(1 – X) = 1 + B(–X)
  = 1 – A(X)
  = 1 – g(p)

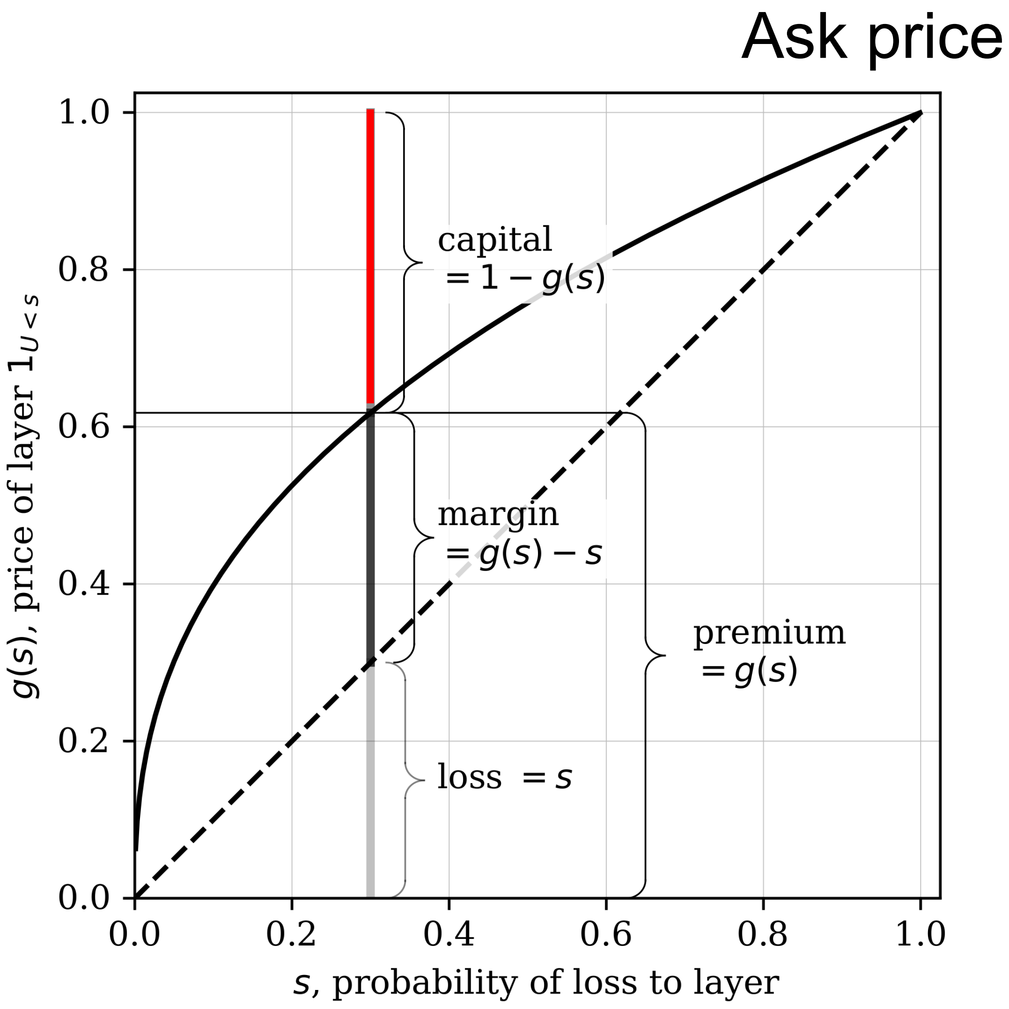
→ sale of residual funds assets of 1

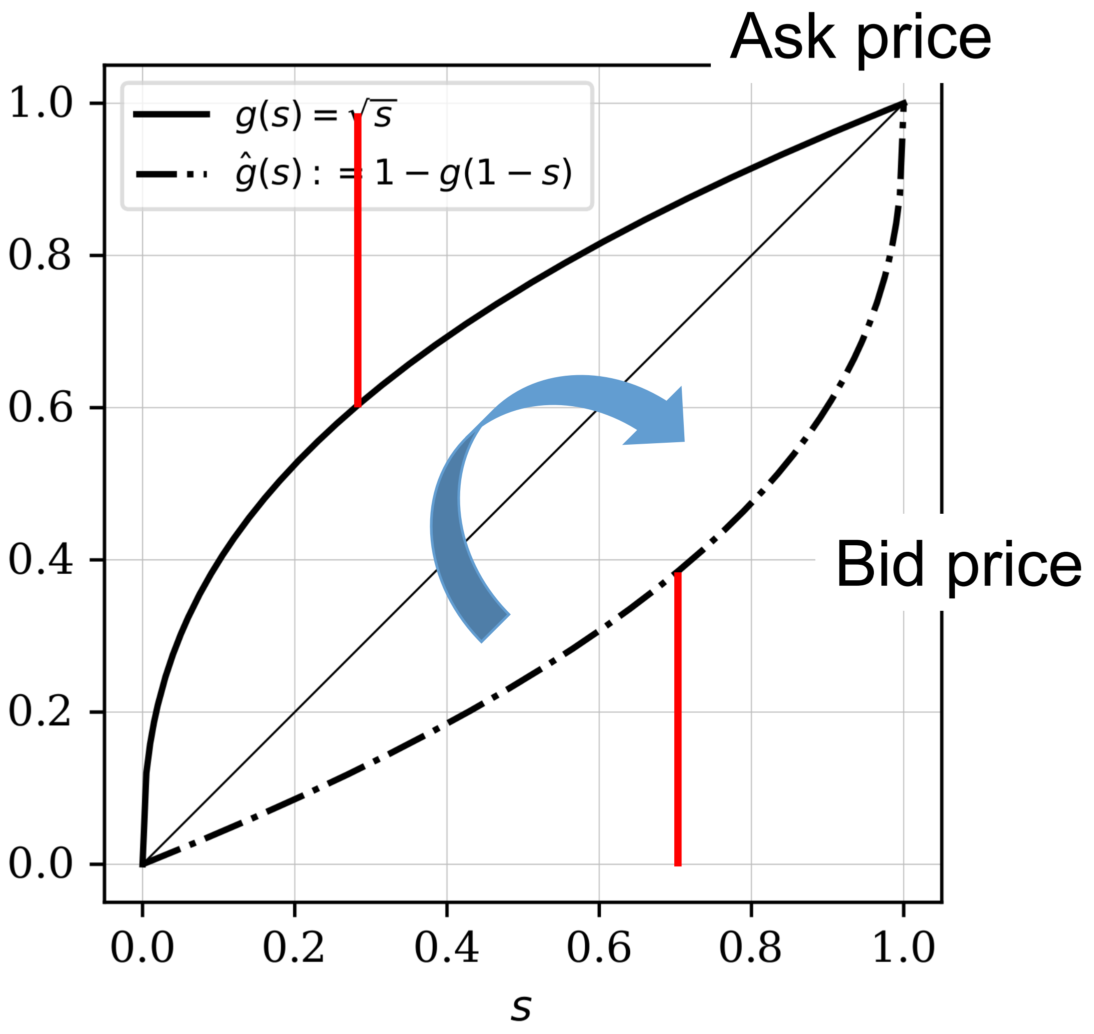
The Premium and Capital Functionals

- **Law invariant** assumption
- s = 1 – *p*
- Premium = A(X) = g(s)
- Capital = B(1 – X) = 1 – g(s)
- Single **distortion function** g determines A and B
- Credit yield curve



The Premium and Capital Functionals





The Story So Far...

- **One** pricing functional for **all** contingent cash flows, but **two** viewpoints
  - Ask price A(X) applies to bought (re)insurance policies, premium
  - Bid price B(X) = –A(–X) applies to sold insurer securities, capital
- Spread allows for underwriter skepticism: Hot/Cold policies both have margins
- Law invariant: price of Ber(*p*) only depends on *p*
- Translation invariance: A(X + c) = A(X) + c for constant c

Next: Price Realistic Portfolio using Bernoulli Risks

- Add-up layer price density
- But A(.) is not additive for all risks...bid-ask spread
- Assume price **additive** for risks that do not diversify against one-another
  – VaR capital adds up
- Comonotonic random variables
  – X = f(Z)
  – Y = g(Z)
  – f, g increasing functions
- Example: two different layers on the same risk

...with these combined assumptions, price of risk equals sum of prices of its Bernoulli layer parts

Pricing Realistic Portfolio from Bernoulli Risks

- $1_{X>x}$ is a Bernoulli risk random variable, with *p* = S(x) := Pr(X>x)
- $1_{X>x}(\omega) = 1$ if $X(\omega) > x$
- All $1_{X>x}$ are comonotonic
- $X = \int 1_{X>x}\,dx$
- Assume ask price functional A is law invariant, comonotonic additive, coherent

Derive price using

- $A(X) = \int A(1_{X>x})\,dx$
- $A(1_{X>x}) = g(S(x))$
- $A(X) = \int g(S(x))\,dx$

Determining the pricing function g

- Three kinds of insurer capital
  - Equity: lowest priority, quota share residual value
  - Debt: tranched, AA, A, BBB,...
  - Reinsurance
    - Catastrophe
    - Aggregate
    - All other

- **Capital structure determines g**

...capital structure plus capital standard determines portfolio price

Adam Smith suggests...

Margin = ($ Amount of Capital) x (Cost / $)

By Unit = ($ **Allocated** Capital) x (Cost / $)

Focus on
capital
allocation

Hard:
assume
constant

Allocate directly using premium functional

Marginal, Euler allocation

- Blessed by economics 101

Natural (e.g., co-TVaR) allocation

- Blessed by finance, risk-adj probs

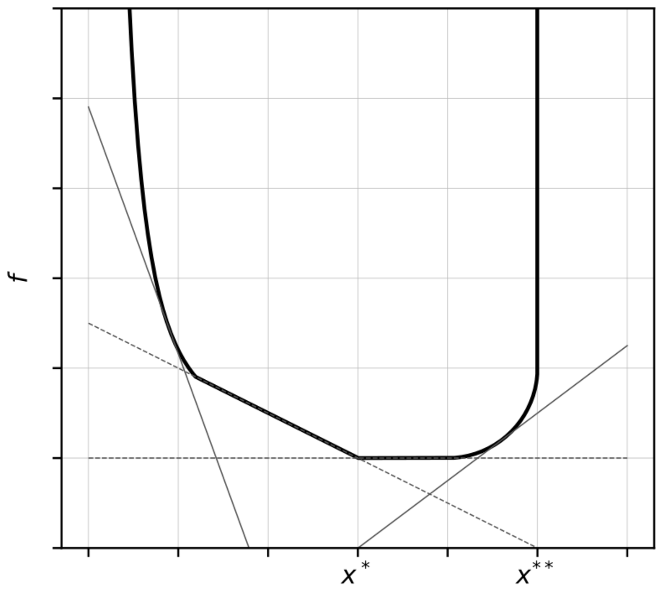
Good news: under general assumptions marginal and natural are the same by **Delbaen's Theorem**

E.g., holds iff all total loss outcomes distinct (no ties) → unique ordering of events

Bad News:

General assumptions do not hold under **limited liability**

- Marginal methods fail because risk measure is not differentiable
- Ordering problem



- Natural allocation not unique
- **Linear Natural Allocation**
  Average over all natural allocations
  = marginal over all orderings

- Allocation always satisfies

  $B(X_i) \le \text{allocation} \le A(X_i)$

- If units are independent

  $E(X_i) \le \text{allocation} \le A(X_i)$

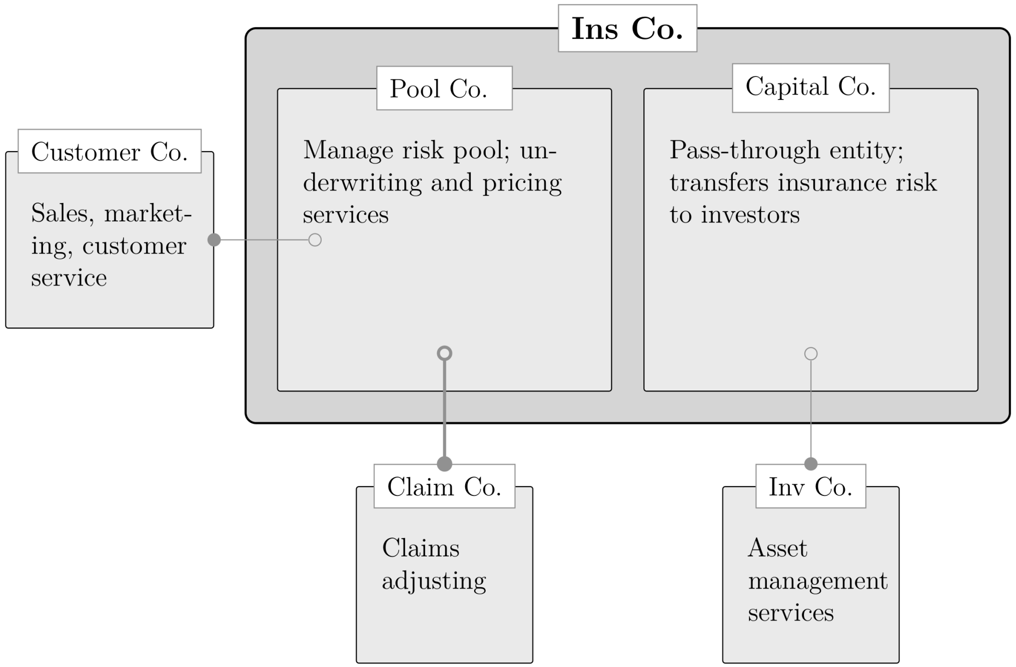
Conclusions

- Calibrate g function to capital structure
- Use binding capital constraint to determine total capital
- Use linear natural allocation to price by unit
- Buying motivation matters!
- Focus on **pricing confidence** and communicating it to investors / owners

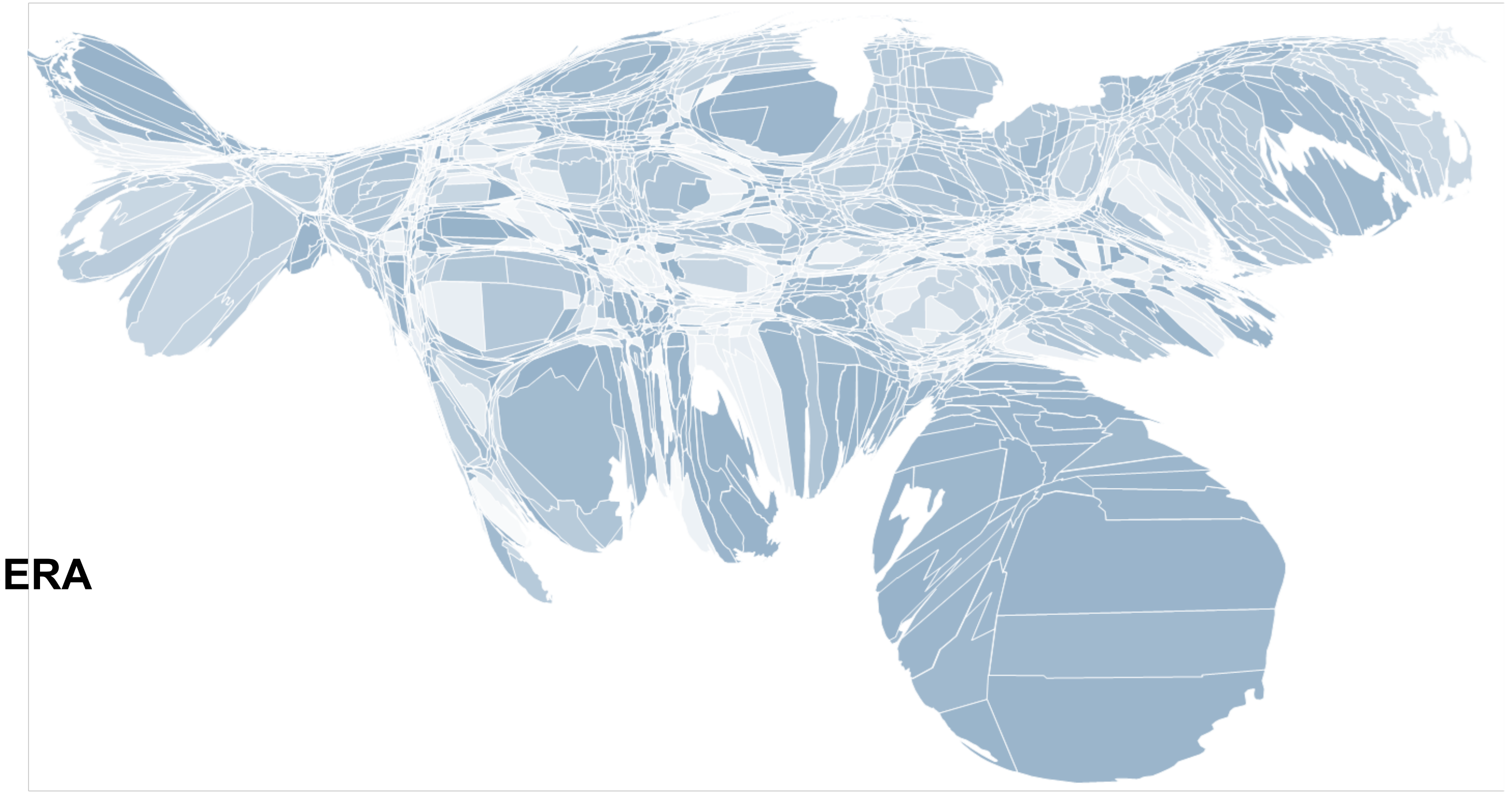
Appendix: Why a return only to capital?

- Capital = all investments made by firm
- Commission includes margin paid to Customer Co. capital
- Fee-for-service options: TPA, MGA
- **Focus: pure "risk premium" net of expenses**

Insurance Company Functions



Contact Information



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Graphic note: County size scaled to RMS loss estimates for hurricane, earthquake and severe weather using Gastner & Newman algorithm